

■QRコードの生成

21 セル×21 セルの誤り訂正レベル Q の生成多項式は,

$$g(x) = (X + \alpha^0)(X + \alpha)(X + \alpha^2)(X + \alpha^3)(X + \alpha^4)(X + \alpha^5)(X + \alpha^6)(X + \alpha^7) \\ (X + \alpha^8)(X + \alpha^9)(X + \alpha^{10})(X + \alpha^{11})(X + \alpha^{12})$$

順次, 各項を乗算すると

$$(X + \alpha^0)(X + \alpha) = X^2 + (\alpha^0 + \alpha)X + \alpha = X^2 + \alpha^{25}X + \alpha$$

$$(X^2 + \alpha^{25}X + \alpha)(X + \alpha^2) = X^3 + (\alpha^{25} + \alpha^2)X^2 + (\alpha + \alpha^{27})X + \alpha^3 = X^3 + \alpha^{198}X^2 + \alpha^{199}X + \alpha^3$$

$$(X^3 + \alpha^{198}X^2 + \alpha^{199}X + \alpha^3)(X + \alpha^3) = X^4 + (\alpha^{198} + \alpha^3)X^3 + (\alpha^{199} + \alpha^{201})X^2 + (\alpha^3 + \alpha^{202})X + \alpha^6 \\ = X^4 + \alpha^{75}X^3 + \alpha^{249}X^2 + \alpha^{78}X + \alpha^6$$

$$(X^4 + \alpha^{75}X^3 + \alpha^{249}X^2 + \alpha^{78}X + \alpha^6)(X + \alpha^4) \\ = X^5 + (\alpha^{75} + \alpha^4)X^4 + (\alpha^{249} + \alpha^{79})X^3 + (\alpha^{78} + \alpha^{253})X^2 + (\alpha^6 + \alpha^{82})X + \alpha^{10} \\ = X^5 + \alpha^{113}X^4 + \alpha^{164}X^3 + \alpha^{166}X^2 + \alpha^{119}X + \alpha^{10}$$

$$(X^5 + \alpha^{113}X^4 + \alpha^{164}X^3 + \alpha^{166}X^2 + \alpha^{119}X + \alpha^{10})(X + \alpha^5) \\ = X^6 + (\alpha^{113} + \alpha^5)X^5 + (\alpha^{164} + \alpha^{118})X^4 + (\alpha^{166} + \alpha^{169})X^3 + (\alpha^{119} + \alpha^{171})X^2 \\ + (\alpha^{10} + \alpha^{124})X + \alpha^{15} \\ = X^6 + \alpha^{166}X^5 + \alpha^0X^4 + \alpha^{134}X^3 + \alpha^5X^2 + \alpha^{176}X + \alpha^{15}$$

$$(X^6 + \alpha^{166}X^5 + \alpha^0X^4 + \alpha^{134}X^3 + \alpha^5X^2 + \alpha^{176}X + \alpha^{15})(X + \alpha^6) \\ = X^7 + (\alpha^{166} + \alpha^6)X^6 + (\alpha^0 + \alpha^{172})X^5 + (\alpha^{134} + \alpha^6)X^4 + (\alpha^5 + \alpha^{140})X^3 + (\alpha^{176} + \alpha^{11})X^2 \\ + (\alpha^{15} + \alpha^{182})X + \alpha^{21} \\ = X^7 + \alpha^{87}X^6 + \alpha^{229}X^5 + \alpha^{146}X^4 + \alpha^{149}X^3 + \alpha^{238}X^2 + \alpha^{102}X + \alpha^{21}$$

$$(X^7 + \alpha^{87}X^6 + \alpha^{229}X^5 + \alpha^{146}X^4 + \alpha^{149}X^3 + \alpha^{238}X^2 + \alpha^{102}X + \alpha^{21})(X + \alpha^7) \\ = X^8 + (\alpha^{87} + \alpha^7)X^7 + (\alpha^{229} + \alpha^{94})X^6 + (\alpha^{146} + \alpha^{236})X^5 + (\alpha^{149} + \alpha^{153})X^4 + (\alpha^{238} + \alpha^{156})X^3 \\ + (\alpha^{102} + \alpha^{245})X^2 + (\alpha^{21} + \alpha^{109})X + \alpha^{28} \\ = X^8 + \alpha^{175}X^7 + \alpha^{238}X^6 + \alpha^{208}X^5 + \alpha^{249}X^4 + \alpha^{215}X^3 + \alpha^{252}X^2 + \alpha^{196}X + \alpha^{28}$$

$$(X^8 + \alpha^{175}X^7 + \alpha^{238}X^6 + \alpha^{208}X^5 + \alpha^{249}X^4 + \alpha^{215}X^3 + \alpha^{252}X^2 + \alpha^{196}X + \alpha^{28})(X + \alpha^8) \\ = X^9 + (\alpha^{175} + \alpha^8)X^8 + (\alpha^{238} + \alpha^{183})X^7 + (\alpha^{208} + \alpha^{246})X^6 + (\alpha^{249} + \alpha^{216})X^5 + (\alpha^{215} + \alpha^2)X^4 \\ + (\alpha^{252} + \alpha^{223})X^3 + (\alpha^{196} + \alpha^5)X^2 + (\alpha^{28} + \alpha^{204})X + \alpha^{36} \\ = X^9 + \alpha^{95}X^8 + \alpha^{246}X^7 + \alpha^{137}X^6 + \alpha^{231}X^5 + \alpha^{235}X^4 + \alpha^{149}X^3 + \alpha^{11}X^2 + \alpha^{123}X + \alpha^{36}$$

$$(X^9 + \alpha^{95}X^8 + \alpha^{246}X^7 + \alpha^{137}X^6 + \alpha^{231}X^5 + \alpha^{235}X^4 + \alpha^{149}X^3 + \alpha^{11}X^2 + \alpha^{123}X + \alpha^{36})(X + \alpha^9) \\ = X^{10} + (\alpha^{95} + \alpha^9)X^9 + (\alpha^{246} + \alpha^{104})X^8 + (\alpha^{137} + \alpha^0)X^7 + (\alpha^{231} + \alpha^{146})X^6 + (\alpha^{235} + \alpha^{240})X^5 \\ + (\alpha^{149} + \alpha^{244})X^4 + (\alpha^{11} + \alpha^{158})X^3 + (\alpha^{123} + \alpha^{20})X^2 + (\alpha^{36} + \alpha^{132})X + \alpha^{45}$$

$$= X^{10} + \alpha^{251}X^9 + \alpha^{67}X^8 + \alpha^{46}X^7 + \alpha^{61}X^6 + \alpha^{118}X^5 + \alpha^{70}X^4 + \alpha^{64}X^3 + \alpha^{94}X^2 + \alpha^{32}X + \alpha^{45}$$

$$\begin{aligned} & (X^{10} + \alpha^{251}X^9 + \alpha^{67}X^8 + \alpha^{46}X^7 + \alpha^{61}X^6 + \alpha^{118}X^5 + \alpha^{70}X^4 + \alpha^{64}X^3 + \alpha^{94}X^2 + \alpha^{32}X + \alpha^{45})(X + \alpha^{10}) \\ &= X^{11} + (\alpha^{251} + \alpha^{10})X^{10} + (\alpha^{67} + \alpha^6)X^9 + (\alpha^{46} + \alpha^{77})X^8 + (\alpha^{61} + \alpha^{56})X^7 + (\alpha^{118} + \alpha^{71})X^6 \\ &\quad + (\alpha^{70} + \alpha^{128})X^5 + (\alpha^{64} + \alpha^{80})X^4 + (\alpha^{94} + \alpha^{74})X^3 + (\alpha^{32} + \alpha^{104})X^2 + (\alpha^{45} + \alpha^{42})X + \alpha^{55} \\ &= X^{11} + \alpha^{220}X^{10} + \alpha^{192}X^9 + \alpha^{91}X^8 + \alpha^{194}X^7 + \alpha^{172}X^6 + \alpha^{177}X^5 + \alpha^{209}X^4 \\ &\quad + \alpha^{116}X^3 + \alpha^{227}X^2 + \alpha^{10}X + \alpha^{55} \end{aligned}$$

$$\begin{aligned} & (X^{11} + \alpha^{220}X^{10} + \alpha^{192}X^9 + \alpha^{91}X^8 + \alpha^{194}X^7 + \alpha^{172}X^6 + \alpha^{177}X^5 + \alpha^{209}X^4 \\ &\quad + \alpha^{116}X^3 + \alpha^{227}X^2 + \alpha^{10}X + \alpha^{55})(X + \alpha^{11}) \\ &= X^{12} + (\alpha^{220} + \alpha^{11})X^{11} + (\alpha^{192} + \alpha^{231})X^{10} + (\alpha^{91} + \alpha^{203})X^9 + (\alpha^{194} + \alpha^{102})X^8 + (\alpha^{172} + \alpha^{205})X^7 \\ &\quad + (\alpha^{177} + \alpha^{183})X^6 + (\alpha^{209} + \alpha^{188})X^5 + (\alpha^{116} + \alpha^{220})X^4 + (\alpha^{227} + \alpha^{127})X^3 + (\alpha^{10} + \alpha^{238})X^2 \\ &\quad + (\alpha^{55} + \alpha^{21})X + \alpha^{66} \\ &= X^{12} + \alpha^{102}X^{11} + \alpha^{43}X^{10} + \alpha^{98}X^9 + \alpha^{121}X^8 + \alpha^{187}X^7 + \alpha^{113}X^6 + \alpha^{198}X^5 \\ &\quad + \alpha^{143}X^4 + \alpha^{131}X^3 + \alpha^{87}X^2 + \alpha^{157}X + \alpha^{66} \end{aligned}$$

$$\begin{aligned} & (X^{12} + \alpha^{102}X^{11} + \alpha^{43}X^{10} + \alpha^{98}X^9 + \alpha^{121}X^8 + \alpha^{187}X^7 + \alpha^{113}X^6 + \alpha^{198}X^5 \\ &\quad + \alpha^{143}X^4 + \alpha^{131}X^3 + \alpha^{87}X^2 + \alpha^{157}X + \alpha^{66})(X + \alpha^{12}) \\ &= X^{13} + (\alpha^{102} + \alpha^{12})X^{12} + (\alpha^{43} + \alpha^{114})X^{11} + (\alpha^{98} + \alpha^{55})X^{10} + (\alpha^{121} + \alpha^{110})X^9 + (\alpha^{187} + \alpha^{133})X^8 \\ &\quad + (\alpha^{113} + \alpha^{199})X^7 + (\alpha^{198} + \alpha^{125})X^6 + (\alpha^{143} + \alpha^{210})X^5 + (\alpha^{131} + \alpha^{115})X^4 + (\alpha^{87} + \alpha^{143})X^3 \\ &\quad + (\alpha^{157} + \alpha^{99})X^2 + (\alpha^{66} + \alpha^{169})X + \alpha^{78} \\ &= X^{13} + \alpha^{74}X^{12} + \alpha^{152}X^{11} + \alpha^{176}X^{10} + \alpha^{100}X^9 + \alpha^{86}X^8 + \alpha^{100}X^7 + \alpha^{106}X^6 + \alpha^{104}X^5 \\ &\quad + \alpha^{130}X^4 + \alpha^{218}X^3 + \alpha^{206}X^2 + \alpha^{140}X + \alpha^{78} \end{aligned}$$

$$\begin{aligned} \mathbf{g}(x) &= X^{13} + \alpha^{74}X^{12} + \alpha^{152}X^{11} + \alpha^{176}X^{10} + \alpha^{100}X^9 + \alpha^{86}X^8 + \alpha^{100}X^7 + \alpha^{106}X^6 + \alpha^{104}X^5 \\ &\quad + \alpha^{130}X^4 + \alpha^{218}X^3 + \alpha^{206}X^2 + \alpha^{140}X + \alpha^{78} \end{aligned}$$